

ON THE COMPLETENESS PROPERTIES OF THE C -COMPACT-OPEN TOPOLOGY ON $C(X)^{1,2}$

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This is a study of the completeness properties of the space $C_{rc}(X)$ of continuous real-valued functions on a Tychonoff space X , where the function space has the C -compact-open topology. Investigate the properties such as completely metrizable, Čech-complete, pseudocomplete and almost Čech-complete.

Keywords: C -compact-open topology, Set-open topology, Čech-complete, Baire space, Function space.

Introduction

The set-open topology on a family λ of nonempty subsets of the Tychonoff space X (the λ -open topology) is a generalization of the compact-open topology and of the topology of pointwise convergence. This topology was first introduced by Arens and Dugundji [7].

All sets of the form $\{f \in C(X) : f(F) \subseteq U\}$, where $F \in \lambda$ and U is an open subset of real line \mathbb{R} , form a subbase of the λ -open topology.

We denote the space $C(X)$ with λ -open topology by $C_\lambda(X)$. Note that the set-open topology and its properties depend on the family λ . So if we take a family λ of all finite, compact or pseudocompact subsets of X then we get point-open, compact-open, pseudocompact-open topology on $C(X)$ respectively. These topologies actively studied and find their application in measure theory and functional analysis.

Sure if we take an arbitrary family λ then a topological space $C_\lambda(X)$ may have weaker properties, for example, it can not be a regular or Hausdorff space.

Special interest for applications when a space $C_\lambda(X)$ is a locally convex topological vector space (TVS). Therefore, we take a “good” family λ of subsets of X which define locally convex TVS on $C(X)$. For example a families of all compact, finite, metrizable compact, sequentially compact, countable compact, pseudocompact or C -compact subsets of X are “good” families (see [18]).

Recall that a subset A of a space X is called *C -compact subset* X if, for any real-valued function f continuous on X , the set $f(A)$ is compact in \mathbb{R} .

Note that, in the case $A = X$, the property of the set A to be C -compact coincides with the pseudocompactness of the space X .

The space $C(X)$, equipped with the set-open topology on the family of all C -compact subsets of X , is denoted by $C_{rc}(X)$.

This article is a continuation of the article [3] on the study of topological properties of the space $C_{rc}(X)$.

The importance of studying the C -compact-open topology on $C(X)$, due to the fact that if $C_\lambda(X)$ is a locally convex TVS then the family λ consists of C -compact subsets of X .

Moreover if $C_\lambda(X)$ is a topological (even paratopological) group then the family λ consists of C -compact subsets of X .

In [19] was found to be characteristic for the space $C_\lambda(X)$ such that $C_\lambda(X)$ is a topological group, TVS or locally convex TVS.

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Note that if the set-open topology coincides with the topology of uniform convergence on the family λ then $C_\lambda(X)$ is a topological algebra.

Recall that the topology of uniform convergence is given by a base at each point $f \in C(X)$. This base consists of all sets $\{g \in C(X) : \sup_{x \in X} |g(x) - f(x)| < \varepsilon\}$. The topology of uniform convergence on elements of a family λ (the λ -topology), where λ is a fixed family of non-empty subsets of the set X , is a natural generalization of this topology. All sets of the form $\{g \in C(X) : \sup_{x \in F} |g(x) - f(x)| < \varepsilon\}$, where $F \in \lambda$ and $\varepsilon > 0$, form a base of the λ -topology at a point $f \in C(X)$. We denote the space $C(X)$ with λ -topology by $C_{\lambda,u}(X)$.

In [19] proved the following theorem (Theorem 3.3).

Theorem 0.1. *For a space X , the following statements are equivalent.*

1. $C_\lambda(X) = C_{\lambda,u}(X)$.
2. $C_\lambda(X)$ is a topological group.
3. $C_\lambda(X)$ is a topological vector space.
4. $C_\lambda(X)$ is a locally convex topological vector space.
5. λ is a family of C -compact sets and $\lambda = \lambda(C)$, where $\lambda(C) = \{A \in \lambda : \text{for every } C\text{-compact subset } B \text{ of the space } X \text{ with } B \subset A, \text{ the set } [B, U] \text{ is open in } C_\lambda(X) \text{ for any open set } U \text{ of the space } \mathbb{R}\}$.

In [3], in addition to studying some basic properties of $C_{rc}(X)$, metrizability, separability and submetrizability of $C_{rc}(X)$ have been studied. In this paper, we study various kinds of completeness of the C -compact topology such as complete metrizability, Čech-completeness, pseudocompleteness and almost Čech-completeness of $C_{rc}(X)$.

Throughout the rest of the paper, we use the following conventions. All spaces are completely regular Hausdorff, that is, Tychonoff.

The elements of the standard subbases of the λ -open topology and λ -topology will be denoted as follows:

$$[F, U] = \{f \in C(X) : f(F) \subseteq U\},$$

$$\langle f, F, \varepsilon \rangle = \{g \in C(X) : \sup_{x \in F} |f(x) - g(x)| < \varepsilon\}, \text{ where } F \in \lambda, U \text{ is an open subset of } \mathbb{R} \text{ and } \varepsilon > 0.$$

If X and Y are any two spaces with the same underlying set, then we use $X = Y$, $X \leq Y$ and $X < Y$ to indicate, respectively, that X and Y have same topology, that the topology Y is finer than or equal to the topology on X and that the topology on Y is strictly finer than the topology on X . The symbols \mathbb{R} and \mathbb{N} denote the spaces of real numbers and natural numbers, respectively.

We recall that a subset of X that is the complete preimage of zero for a certain function from $C(X)$ is called a zero-set. A subset O of a space X is called functionally open (or a cozero-set) if $X \setminus O$ is a zero-set.

Cover is called functionally open if it consists of functionally open subsets of X .

Let $G \subseteq C(X)$. A set $A \subseteq X$ is said to be G -bounded if $f(A)$ is a bounded subset of \mathbb{R} for each $f \in G$. We say that A is bounded in X if A is G -bounded for $G = C(X)$.

A space X is called a μ -space if every closed bounded subset of X is compact. In the literature, a μ -space is also called a hyperisocompact or a Nachbin-Shirota space (NS -space for brevity).

The closure of a set A will be denoted by \bar{A} ; the symbol \emptyset stands for the empty set.

If $A \subseteq X$ and $f \in C(X)$, then we denote by $f|_A$ the restriction of the function f to the set A . As usual, $f(A)$ and $f^{-1}(A)$ are the image and the complete preimage of the set A under the mapping f , respectively.

The constant zero function defined on X is denoted by f_0 . We call it the constant zero function in $C(X)$.

The remaining notation can be found in [5].

Obvious that a pseudocompact subset of X is a C -compact subset of X and a C -compact subset of X is a bounded subset of X by definition.

In [5] given a well-known Isbell-Frolík-Mrówka space in which the concepts of pseudocompactness and C -compactness differ even for closed subsets.

We note some important properties of C -compact subset (see [2] and [8]).

The subset A is an C -compact subset of X if and only if every countable functionally open (in X) cover of A has a proper subcollection whose union is dense in A .

For any Tychonoff space X , pseudocompactness equivalent to feebly compactness of X . Recall that a space X is called a feebly compact if whenever countably infinite locally finite open cover of X has a proper subcollection whose union is dense in X . It is well known that the closure of the pseudocompact (bounded) subset of X will be pseudocompact (bounded) subset of X . It holds true for C -compact set [1].

Note that for a closed subset A in a normal Hausdorff space X , the following equivalent (see [14]).

1. A is countably compact.
2. A is pseudocompact.
3. A is C -compact subset of X .
4. A is bounded.

Recall that a Tychonoff space X is called submetrizable if X admits a weaker metrizable topology.

Note that for a subset A in a submetrizable space X , the following are equivalent (see [4]).

1. A is countably compact subset of X .
2. A is pseudocompact subset of X .
3. A is sequentially compact subset of X .
4. A is C -compact subset of X .
5. A is compact subset of X .
6. A is metrizable compact subset of X .

Note that every closed bounded subset of Dieudonné complete space is compact (see [14]).

1. Uniform Completeness of $C_{rc}(X)$

There are three ways to consider the C -compact-open topology on $C(X)$ [3].

First, one can use as subbase the family $\{[A, V] : A \text{ is a } C\text{-compact subset of } X \text{ and } V \text{ is an open subset of } \mathbb{R}\}$. But one can also consider this topology as the topology of uniform convergence on the C -compact subsets of X , in which case the basic open sets will be of the form $\langle f, F, \varepsilon \rangle$, where $f \in C(X)$, F is a C -compact subset of X and ε is a positive real number.

The third way is to look at the C -compact-open topology as a locally convex topology on $C(X)$. For each C -compact subset A of X and $\varepsilon > 0$, we define the seminorm p_A on $C(X)$ and $V_{A,\varepsilon}$ as follows: $p_A(f) = \sup\{|f(x)| : x \in A\}$ and $V_{A,\varepsilon} = \{f \in C(X) : p_A(f) < \varepsilon\}$. Let $\Psi = \{V_{A,\varepsilon} : A \text{ is a } C\text{-compact subset of } X, \varepsilon > 0\}$. Then for each $f \in C(X)$, $f + \Psi = \{f + V : V \in \Psi\}$ forms a neighborhood base at f . This topology is locally convex since it is generated by a collection of seminorms and it is same as the C -compact-open topology on $C(X)$. It is also easy to see that this topology is Tychonoff.

The topology of uniform convergence on the C -compact subsets of X is actually generated by the uniformity of uniform convergence on these subsets. Recall that a uniform space E is called complete provided that every Cauchy net in E converges to some element in E .

In order to characterize the uniform completeness of $C_{rc}(X)$, we need to talk about rc -continuous functions and rc_f -spaces.

Definition 1.1. A function $f : X \mapsto \mathbb{R}$ is said to be rc -continuous, if for every C -compact subset $A \subseteq X$, there exists a continuous function $g : X \mapsto \mathbb{R}$ such that $g|_A = f|_A$. A space X is called a rc_f -space if every rc -continuous function on X is continuous.

Theorem 1.1. *The space $C_{rc}(X)$ is uniformly complete if and only if X is a rc_f -space.*

Proof. Note that a C -compact subset of X is a bounded set. So by Theorem 4.6 (see [14]), $C_{rc}(X)$ is uniformly complete. \square

2. Complete metrizability and some related completeness properties of $C_{rc}(X)$

In this section, we study various kinds of completeness $C_{rc}(X)$. In particular, here we study the complete metrizability of $C_{rc}(X)$ in a wider setting, more precisely, in relation to several other completeness properties.

A space X is called Čech-complete if X is a G_δ -set in βX . A space X is called locally Čech-complete if every point $x \in X$ has a Čech-complete neighborhood. Another completeness property which is implied by Čech-completeness is that of pseudocompleteness.

This is space having a sequence of π -bases $\{\mathcal{B}_n : n \in \mathbb{N}\}$ such that whenever $B_n \in \mathcal{B}_n$ for each n and $\overline{B_{n+1}} \subseteq B_n$, then $\bigcap \{B_n : n \in \mathbb{N}\} \neq \emptyset$ (see [20]).

In [6], it has been shown that a space having a dense Čech-complete subspace is pseudocomplete and a pseudocomplete space is a Baire space.

Let \mathcal{F} and \mathcal{U} be two collections of subsets of X . Then \mathcal{F} is said to be controlled by \mathcal{U} , if for each $U \in \mathcal{U}$, there exists $F \in \mathcal{F}$ such that $F \subseteq U$. A sequence $\{U_n\}$ of subsets of X is said to be complete if every filter base \mathcal{F} on X which is controlled $\{U_n\}$ clusters at some $x \in X$. A sequence $\{\mathcal{U}_n\}$ of collections of subsets of X is called complete if $\{U_n\}$ is a complete sequence of subsets of X whenever $U_n \in \mathcal{U}_n$ for all $n \in \mathbb{N}$. It has been shown in [11, Theorem 2.8] that the following statements are equivalent for a Tychonoff space X :

- (1) X is a G_δ -subset of any Hausdorff space in which it is densely embedded;
- (2) X has a complete sequence of open covers;
- (3) X is Čech-complete.

From this result, it easily follows that a Tychonoff space X is Čech-complete if and only if X is a G_δ -subset of any Tychonoff space in which it is densely embedded.

We call a \mathcal{U} of subsets of X an almost-cover of X if $\bigcup \mathcal{U}$ is dense in X . We call a space almost Čech-complete if X has a complete sequence of open almost-covers. Every almost Čech-complete space is a Baire space, see [16, Proposition 4.5].

The property of being a Baire space is the weakest one among the completeness properties we consider here. Since $C_{rc}(X)$ is a locally convex space, $C_{rc}(X)$ is a Baire space if and only if $C_{rc}(X)$ is of second category in itself. Also since a locally convex Baire space is barreled, first we find a necessary condition for $C_{rc}(X)$ to be barreled. A locally convex space L is called barreled if each barrel in L is a neighborhood of 0_L .

Theorem 2.1. *If $C_{rc}(X)$ is barreled, then every bounded subset of X is contained in a C -compact subset of X .*

Proof. Let A be a bounded subset of X and let $W = \{f \in C(X) : p_A(f) \leq 1\}$. Then it is routine to check that W is closed, convex, balanced and absorbing, that is, W is a barrel in $C_{rc}(X)$. Since $C_{rc}(X)$ is barreled, W is a neighborhood of f_0 and consequently there exist a closed C -compact subset P of X and $\varepsilon > 0$ such that $\langle f_0, P, \varepsilon \rangle \subseteq W$. We claim that $A \subseteq P$. If not, let $x_0 \in A \setminus P$. So there exists a continuous function $f : X \mapsto [0, 2]$ such that $f(x) = 0$ for all $x \in P$ and $f(x_0) = 2$. Clearly $f \in \langle f_0, P, \varepsilon \rangle$, but $f \notin W$. Hence we must have $A \subseteq P$. \square

If X is μ -space, then every closed bounded (C -compact) subset of X is a compact and consequently the C -compact-open and compact-open topologies on $C(X)$ coincide. But by famous Nachbin-Shirota theorem, $C_c(X)$ is barreled if X is μ -space. Hence if X is realcompact, then $C_{rc}(X)$ is barreled. In particular, since the Niemytzki plane L is realcompact, $C_{rc}(L)$ is barreled.

But there are space X such that $C_{rc}(X)$ is not barreled.

E x a m p l e 1. (Dieudonné Plank) Let $X = [0, \omega_1] \times [0, \omega_0] \setminus \{(\omega_1, \omega_0)\}$.

Topology τ is generated from the base: all the points of set $[0, \omega_1) \times [0, \omega_0)$ are isolated, and sets of the form $U_\alpha(\beta) = \{(\beta, \gamma) : \alpha < \gamma \leq \omega_0\}$ and $V_\alpha(\beta) = \{(\gamma, \beta) : \alpha < \gamma \leq \omega_1\}$.

Let $A = \{(\omega_1, n) : 0 \leq n < \omega_0\}$. Take an arbitrary C -compact subset B of the space X . Since a set $\{\alpha\} \times [0, \omega_0]$ is a clopen (and hence functionally open) for any $\alpha < \omega_1$, then set $([0, \omega_1] \times \{\beta\}) \cap B$ consists of more than finite number of points for any $\beta \leq \omega_0$.

It follows that B is a compact subset of X . In [14] was proved that the set A is a closed bounded subset of X . Since A is not compact subset of X and each C -compact subsets of X is a compact then $C_{rc}(X)$ is not barreled.

Recall that a space X is called hemi- C -compact if there exists a sequence of C -compact subsets $\{A_n : n \in \mathbb{N}\}$ in X such that for any C -compact subset A of X , $A \subseteq A_{n_0}$ holds for some $n_0 \in \mathbb{N}$ [3].

In [3] obtained the characterization of metrizability of space $C_{rc}(X)$.

Theorem 2.2. *For any space X , the following are equivalent.*

1. $C_{rc}(X)$ is metrizable.
2. $C_{rc}(X)$ is of first countable.
3. $C_{rc}(X)$ is of countable type.
4. $C_{rc}(X)$ is of pointwise countable type.
5. $C_{rc}(X)$ has a dense subspace of pointwise countable type.
6. $C_{rc}(X)$ is an M -space.
7. $C_{rc}(X)$ is a q -space.
8. X is hemi- C -compact.

The following theorem gives a characterization of complete metrizable of the space $C_{rc}(X)$.

Theorem 2.3. *For any space X , the following assertions are equivalent.*

1. $C_{rc}(X)$ is a completely metrizable.
2. $C_{rc}(X)$ is Čech-complete.
3. $C_{rc}(X)$ is locally Čech-complete.
4. $C_{rc}(X)$ is an open continuous image of a paracompact Čech-complete space.
5. $C_{rc}(X)$ is an open continuous image of a Čech-complete space.
6. X is a hemi- C -compact rc_f -space.

P r o o f. We have earlier noted that $C_{rc}(X)$ is completely metrizable if and only if it is uniform complete and metrizable. Hence by Theorem 1.1 and by Theorem 2.2, (1) \Leftrightarrow (6). Note that (1) \Rightarrow (2) \Rightarrow (3) and (1) \Rightarrow (4) \Rightarrow (5). Also (3) \Rightarrow (5), see [5, 3.12.19.(d)].

(5) \Rightarrow (1) A Čech-complete space is of pointwise countable type and the property of being pointwise countable type is preserved by open continuous maps. Hence $C_{rc}(X)$ is of pointwise countable type and consequently by Corollary 5.2 [3], $C_{rc}(X)$ is metrizable and hence $C_{rc}(X)$ is paracompact. So by Pasynkov's theorem [5, Theorem 5.5.8 (b)], $C_{rc}(X)$ is Čech-complete. But a Čech-complete metrizable space is completely metrizable. \square

Note that proof of Theorem 2.3 is similar to that of Theorem 3.3 in [15] on a complete metrizable of space $C(X)$ with the pseudocompact-open topology.

For studying the properties of pseudocomplete and almost Čech-complete we need to embed the space $C_{rc}(X)$ in a larger locally convex function space.

Let $RC(X) = \{f \in \mathbb{R}^X : f|_A \text{ is continuous for each } C\text{-compact subset } A \text{ of } X\}$. As in case of $C_{rc}(X)$, we can define C -compact-open topology on $RC(X)$. In particular, this is a locally convex Hausdorff topology on $RC(X)$ generated by the family of seminorms $\{p_A : A \text{ is a } C\text{-compact subset of } X\}$, where for $f \in RC(X)$, $p_A(f) = \sup\{|f(x)| : x \in A\}$.

We denote the space $RC(X)$ with the C -compact-open topology by $RC_{rc}(X)$. It is clear that $C_{rc}(X)$ is a subspace of $RC_{rc}(X)$. Moreover the proof of the following result is immediate.

Theorem 2.4. *If every closed C -compact subset of X is C -embedded in X , then $C(X)$ is dense in $RC_{rc}(X)$.*

In next theorem, the term σ -space refers to a space having a σ -locally finite network. Every metrizable space is a σ -space.

Theorem 2.5. *For a space X , consider the following conditions.*

1. $C_{rc}(X)$ is completely metrizable.
2. $C_{rc}(X)$ is a pseudocomplete σ -space.
3. $C_{rc}(X)$ is a pseudocomplete q -space.
4. $C_{rc}(X)$ contains a dense completely metrizable subspace.
5. $C_{rc}(X)$ contains a dense Čech-complete subspace.
6. $C_{rc}(X)$ is almost Čech-complete.

Then $(1) \Rightarrow (2) \Rightarrow (3) \Rightarrow (4) \Rightarrow (5) \Leftrightarrow (6)$

P r o o f. $(1) \Rightarrow (2)$ and $(4) \Rightarrow (5)$. These are immediate.

$(2) \Rightarrow (3)$. A Baire space, which is a σ -space as well, has a dense metrizable subspace, see [43]. So if $C_{rc}(X)$ is a pseudocomplete σ -space, then it contains a dense metrizable space. Since every metrizable space is of pointwise countable type, by Theorem 2.2, $C_{rc}(X)$ is a q -space.

$(3) \Rightarrow (4)$ If $C_{rc}(X)$ is a q -space, then by Theorem 2.2, $C_{rc}(X)$ is metrizable. But a metrizable space is pseudocomplete if and only if it contains a dense completely metrizable subspace, see [6, Corollary 2.4.].

$(5) \Leftrightarrow (6)$ follows from [16, Propositions 4.4, 4.7]. □

R e m a r k. If $C_{rc}(X)$ is only assumed to be pseudocomplete, it may not be almost Čech-complete. Let S be an uncountable space in which all points are isolated except for a distinguished point s , a neighborhood of s being any set containing s whose complement is countable [12, 4N].

It can be easily shown that S is a normal space. Since every C -compact subset of S is finite, the C -compact-open topology on $C(S)$ coincides with the point-open topology on $C(S)$. Since S is uncountable, $C_p(S)$ is not metrizable. But $C_p(S)$ is pseudocomplete, since every countable subset in a P -space is closed. But since $C_p(X)$ is not metrizable, by Theorem 5.7 [13], it is not almost Čech-complete either.

Theorem 2.6. *If every closed C -compact subset X is C -embedded in X , then the following assertions are equivalent.*

1. $C_{rc}(X)$ is completely metrizable.
2. $C_{rc}(X)$ is almost Čech-complete.
3. X is a hemi- C -compact rc_f -space.

P r o o f. We only need to show that $(2) \Rightarrow (3)$. If $C_{rc}(X)$ is almost Čech-complete, then $C_{rc}(X)$ contains a dense Čech-complete subspace G . Since every closed C -compact subset of X is C -embedded in X , $C(X)$ is dense in $RC_{rc}(X)$ and consequently G is dense in $RC_{rc}(X)$. Now since $RC_{rc}(X)$ contains a dense Baire subspace G , $RC_{rc}(X)$ is itself a Baire space. Also since G is Čech-complete, G is a G_δ -set in $RC_{rc}(X)$.

Note that every rc -continuous function on X is in $RC_{rc}(X)$. In order to show that X is a rc_f -space, we will show that $RC(X) = C(X)$. So let $f \in RC(X)$. Define the map $T_f : RC_{rc}(X) \mapsto$

$RC_{rc}(X)$ by $T_f(g) = f + g$ for all $g \in RC(X)$. Since $RC_{rc}(X)$ is a locally convex space, T_f is a homeomorphism and consequently $T_f(G)$ is a dense G_δ -subset of $RC_{rc}(X)$. Since $RC_{rc}(X)$ is a Baire space, $G \cap T_f(G) \neq \emptyset$. Let $h \in G \cap T_f(G)$. Then there exists $g \in G$ such that $h = f + g$. So $f = g - h \in C(X)$. \square

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